

# Frege's *Foundations of Arithmetic*

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## Business

- Please send an email to [jeffrey.russell@philosophy.ox.ac.uk](mailto:jeffrey.russell@philosophy.ox.ac.uk) with the subject line “Frege” so I can put your email address on my list for any class announcements.
- I'll make this syllabus and other materials available at <http://users.ox.ac.uk/~sfop0399/> under “Teaching”.

## Lecture plan

Please note that the “further reading” items are not a tutorial reading list. Some of the readings listed may be on your reading lists, but many of them are on related issues that go beyond the main content of the course, in case you are interested.

The section numbers listed for each week are the parts of *FA* that we'll focus on. It will probably be helpful to re-read those parts beforehand so you have the material fresh in your mind.

**Week 1.** Why *FA* is worth our time. What Frege aimed to achieve. Is it possible to prove  $2 + 2 = 4$ ?

Introduction, §§1–6, 87–91, 105

*Further reading:* Boolos, “Gottlob Frege and the Foundations of Arithmetic”. Russell, *Introduction to Mathematical Philosophy*, *The Principles of Mathematics*.

**Week 2.** Are the truths of arithmetic empirical? Do number-concepts depend on experience? Are numbers properties of physical things?

§§7–11, 17, 21–25

*Further reading:* Quine, “Two Dogmas of Empiricism”. Maddy, “Believing the axioms”.

**Week 3.** Three neat ideas: (1) “The content of a statement of number is an assertion about a concept.” (2) “If we are to use the symbol  $a$  to signify an object, we must have a criterion for deciding in all cases whether  $b$  is the same as  $a$ , even if it is not always in our power to apply this criterion.” (3) “When two numbers are so combined as that the one has always an unit answering to every unit of the other, we pronounce them equal.” (Hume’s Principle)

§§45–48, 53–65

**Week 4.** Proof of Frege’s Theorem: the laws of arithmetic are logical consequences of Hume’s Principle. Leibniz’s proof that  $2+2=4$  repaired. Infinite cardinals.

§70–72, 74–91

*Further reading:* Exercises to be posted to my website. Stanford Encyclopedia: <http://plato.stanford.edu/entries/frege-theorem/>. Heck, *Frege’s Theorem*.

**Week 5.** Frege’s objections to treating HP as a basic postulate. Frege’s definition of “the number of  $F$ ’s” and his proof of HP. The nature of definitions and analytic truths.

§§62–73, 92–104

*Further reading:* Boolos and Wright, “Is Hume’s Principle analytic?” Quine, “Carnap on logical truth”.

**Weeks 6 and 7.** Beyond *FA*. Russell’s paradox. The logic in logicism. Could HP be a logical truth? How much of Frege’s project is undermined by the paradox? The “bad company” objection and consistency proofs. Gödel’s Theorem.

§§87–91, 94

*Further reading:* Boolos, “Nominalist Platonism”, “Gödel’s Second Incompleteness Theorem explained in words of one syllable”. Benacerraf, “What numbers could not be”, “Mathematical truth”.

**Week 8.** Loose ends & revision