# The Limits of Logic (Phil 450, Fall 2015)

Monday and Wednesday 12:00–1:30pm
Waite Phillips Hall 204

Jeff Russell
jeff.russell@usc.edu

## Office Hours

Stonier 227
Wednesdays 4–5:30pm
Some Fridays 1:30–2:45pm (subject to change)
Or by appointment

**Please come to office hours** if you feel like you could use help with problems or more discussion of what's going on.

## Goals

- Develop tools for thinking carefully about languages, arguments, representation, truth, computing, proof, and their limitations.
- Get really good at reading, writing, and presenting precise arguments.
- Learn some beautiful truths, which have been entrusted to us as philosophers to carry to the world.

## Evaluation

The only way to learn logic is by doing logic. You'll be responsible not just for taking in the content that I'll teach, but for discovering ideas and teaching them to others.

- **Most important:** Problems presented in class (30%)
  You should plan to present roughly one problem each week. *See the description below.*
- Participation (5%)
  Be present, be on time, ask questions, and be helpful to others.
- Problem-solving wiki (5%)
  *See the description below.*
- Writing assignment (20%)
  *See the separate handout.*
- Midterm take-home exam, due October 28, TBC (20%)
- Final take-home exam, due December 16, TBC (20%)

# The Problem-Solving Wiki

Over the course of the semester, we're going to build up a toolbox of techniques for solving problems in formal logic. In order to help us remember and apply these ideas, we're going to write up descriptions of strategies and techniques in a shared online document. This document is hosted here:

https://logicproblemsolving.wikispaces.com

You can join the wiki in order to make contributions using this link:

https://wikispaces.com/join/4TQDN94

This link expires August 29, so make sure to join by the end of the week!

We'll also use the wiki for assigning problems for presentations before each class. You can also feel free to use the discussion page to talk to each other about the class.

There are two ways to contribute to the wiki. One is to add a new strategy: add a page with a helpful label for the strategy, and then explain in general terms when the strategy is applicable, and how to use it. The other is to add an example to an existing page, which helps illustrate how to use a strategy.

I've already posted some strategies to help get us started.

This idea is based on the "Tricki", which is being put together by mathematicians. (http://www.tricki.org) This is pretty incomplete so far, but already includes some helpful ideas. (But it doesn't have much logic, yet.) For example:

http://www.tricki.org/article/General_problem-solving_tips

One way to contribute is to find a useful idea from the Tricki, explain it in a way that makes sense for our class, and provide an example using material from our course. If you do use content or an idea from the Tricki, make sure to **cite your source** by providing a link.

(This is my first time using wikispaces, so we may revise this plan if we encounter technical difficulties. We'll also see if we find other helpful uses for it.)

# Presentations

Roughly every other class meeting will be devoted to student presentations of solutions to problems. The problems are a central part of the course content. You should make a serious attempt at all of the exercises. (It will be hard to keep up with the ideas without doing this.) You should plan to present roughly one problem each week (or two).

When you present a problem, you are the teacher for that part of the class. Your goal should be to help everyone in the class understand the main ideas. Most of the problems are proofs: your job is to clearly justify a statement, in a way that convinces everyone that the statement is true, and also (ideally) illuminates *why* it's true.

- You should plan ahead what things you'll need to write on the board. If you expect you'll be one of the first few people to present, try to arrive a few minutes early so you can write things up.
- You should start by explaining what the problem says, and what it means, if there is anything about it that isn't totally clear.
- You should carefully explain your reasoning and justification.
- You should pay particular attention to places where it wasn't obvious how the argument should go. Did you use any tricks? You might also discuss alternative strategies that didn't work out. Basically, think about what would help someone else figure out how to do similar problems.
- You don't always need to include every step of the justification, but if there are any gaps, you should be prepared to answer questions about how to fill them in.
- If you think you've said enough, don't feel like you have to pad it out. Short is good.

The problems vary a lot. Sometimes a single sentence will be an adequate answer. In other cases, a good presentation may take ten minutes or so. Hopefully everyone will get a chance to present a variety of different kinds of problems through the semester.

## Collaboration

I encourage you to work together on exercises, but you are individually responsible for understanding, explaining, and defending anything you present in class. It's easy to trick yourself into thinking you understand something better than you do. This helps nobody. Here's a guideline if you work on a problem with others: set aside your notes from group sessions and write up your final solution on your own.

This does not apply to take-home exams. Do those by yourself.

## Texts

Our main text will be notes that I am writing. This is a work in progress, and it will be evolving as the semester progresses. (So I don't recommend printing it all out!) I'll post these notes on the class wiki.

Here are some suggestions for some alternative resources to look at. (If you come across other useful resources, let me know!) On the course syllabus I'll also list relevant parts of textbooks for further reading.

- Sider, *Logic for Philosophy*
- Boolos, Burgess, and Jeffrey, *Computability and Logic*
- Enderton, *Mathematical Logic*
- The Open Logic Project: http://openlogicproject.org/download/

Tim Gowers has a series of blog posts on basic logical reasoning. These are oriented towards *using* logic in order to solve mathematical problems.

https://gowers.wordpress.com/2011/10/09/basic-logic-summary/
https://gowers.wordpress.com/category/cambridge-teaching/basic-logic/

## Outline of Topics

1. Sets and Functions
    - No set has as many members as it has subsets. (Cantor's Theorem)
    - *See also* Sider 1.8; Enderton 0.
2. The Infinite and the Uncountable
    - We can prove things about all numbers by taking for granted things about smaller numbers, and we can define functions on numbers using *recursion*.
    - There are just as many finite sequences as counting numbers.
    - There are more infinite sequences than counting numbers. (Cantor's Theorem 2)
    - *See also* BBJ 1 & 2; Enderton 0.
3. Structures
4. First-Order Logic

- *See also* Sider 4 & 5; BBJ 9 & 10; Enderton 2.0–2.2; Russell, "On Denoting."

5. The Inexpressible
    - No consistent theory can adequately describe itself. (Tarski's Theorem)
    - *See also* BBJ 17 & 15. Tarski, "The Semantic Conception of Truth and the Foundations of Semantics".
6. The Undecidable
    - There is no systematic method for finding out which methods will succeed. (Turing's Theorem)
    - There is no systematic method for finding out which sentences are consistent in first-order logic. (Church's Theorem)
    - *See also* BBJ 6, 7, 4.1, 16; Enderton ch. 3.
7. The Unprovable
    - No theory is strong, simple, consistent, and complete. (Gödel's Incompleteness Theorem)
    - No consistent theory proves its own consistency. (Gödel's Second Incompleteness Theorem)
    - *See also* BBJ 17, 18. Enderton ch. 3. Boolos, "Gödel's Second Incompleteness Theorem Explained in Words of One Syllable."
8. The Indescribable
    - Every argument in first-order logic has a proof or a counterexample, but not both. (Completeness)
    - No valid argument in first-order logic essentially uses infinitely many premises. (Compactness)
    - Any consistent theory in first-order logic is consistent with there only being countably many things. (The Löwenheim-Skolem Theorem)
    - "There are only finitely many $F$'s" cannot be expressed in first-order logic.
    - *See also* Sider 2.9; Enderton 2.5; BBJ ch. 13.
9. Second-Order Logic (if we have time)

## Equality

This classroom is a safe space. Discrimination on the basis of race, gender, sexuality, religion, age, or other identities is unacceptable. If at any time while at USC you feel you have experienced harassment or discrimination, you can file a complaint: see http://equity.usc.edu for more information. You are also welcome to bring the complaint to any faculty or staff member at USC.

## Students with Disabilities

Any student requesting academic accommodations based on a disability is required to register with Disability Services and Programs (DSP) each semester. A letter of verification for approved accommodations can be obtained from DSP. Please be sure the letter is delivered to me as early in the semester as possible.

**Disability Services and Programs**
Located in STU 301
Open 8:30am–5pm, Monday–Friday
213-740-0776 (Phone)
213-740-6948 (TTD)
ability@usc.edu
http://sait.usc.edu/academicsupport/centerprograms/dsp/home_index.html

## Academic Integrity

USC seeks to maintain an optimal learning environment. General principles of academic honesty include the concept of respect for the intellectual property of others, the expectation that individual work will be submitted unless otherwise allowed by an instructor, and the obligations both to protect one's own academic work from misuse by others as well as to avoid using another's work as one's own. All students are expected to understand and abide by these principles. Scampus, The Student Guidebook, contains the Student Conduct Code in Section 11.00, while the recommended sanctions are located in Appendix A: http://usc.edu/dept/publications/SCAMPUS/gov/ Students will be referred to the Office of Student Judicial Affairs and Community Standards for further review should there be any suspicion of academic dishonesty. The Review process can be found at: http://usc.edu/student-affaris/SJACS/ Information on intellectual property at USC is available at: http://usc.edu/academe/acsen/issues/ipr/index.html

## Emergencies

In case of emergency, when travel to campus is difficult, if not impossible, USC executive leadership will announce a digital way for instructors to teach students in their residence halls or homes using a combination of the Blackboard LMS (Learning Management System), teleconferencing, and other technologies. Instructors should be prepared to assign students a "Plan B" project that can be completed "at a distance." For additional information about maintaining your classes in an emergency, please access: http://cst.usc.edu/services/emergencyprep.html

## Changes

I may change anything at any time.