The Limits of Logic (Phil 450, Spring 2017)

Jeff Russell (jeff.russell@usc.edu)

Tuesday and Thursday 9:30–10:50am
Room TBA

Office Hours

Stonier 227, Tuesday and Thursday 12:30–1:30pm, or by appointment

Please come to office hours or make an appointment if you feel like you could use help with problems or more discussion of what's going on.

Goals

- Develop conceptual and technical tools for thinking carefully about language, reasoning, representation, truth, computing, and their limitations.
- Get really good at reading, writing, and presenting precise arguments.
- Learn some beautiful truths, which have been entrusted to us as philosophers to carry to the world.

Evaluation

The only way to learn logic is by doing logic. You'll be responsible not just for taking in the content that I'll teach, but for discovering ideas and teaching them to others.

- Most important: Problems presented in class (35%)
  You should plan to present roughly one problem each week. *See the description below*.
- Participation (5%)
  Be present, be on time, ask questions, and be helpful to others.
- Writing assignment, due April 28, TBC (15%)
  See the separate handout.
- In-class midterm exam, Thursday, March 2 (20%)
- Take-home final exam, due May 5, TBC (25%)

Presentations

Roughly every other class meeting will be devoted to student presentations of solutions to exercises. The exercises are a central part of the course content. You should make a serious attempt at all of the exercises, though you shouldn't worry if you don't solve all of them. It will be hard to keep up with the ideas without thinking hard about the exercises. You should plan to present roughly one exercise each week.

When you present a problem, you are the teacher for that part of the class. Your goal should be to help everyone in the class understand the main ideas involved in your problem. Most of the problems are proofs: your job is to clearly justify a statement, in a way that convinces everyone that the statement is true, and also (ideally) illuminates *why* it's true. When it's appropriate, you should also try to explain how the problem fits with the rest of what we're doing, and why it might be important.

- You should sign up online to present a problem before class. I will distribute a link to the sign-up page.
- You should plan ahead what things you'll need to write on the board. If you'll be presenting, it's very helpful if you can arrive at class a few minutes early so you can write things up.
- You should always start by explaining what the problem says, and what it means. Is there anything about the problem that wasn't totally clear?
- You should carefully explain your reasoning and justification. You'll get lots of practice through the semester seeing how this goes for technical reasoning. Don't worry if at the beginning of the semester you don't have the hang of exactly what you need to include in your explanation; this will come with more experience.
- You should pay particular attention to places where it wasn't obvious how the argument should go. These are the parts that are most helpful to explain to others. Did you use any special tricks? You might also discuss alternative problem-solving strategies you tried that didn't work out. Basically, think about what would help someone else figure out how to do similar problems.
- You don't need to include every step of the justification, but if there are any gaps, you should be prepared to answer questions about how to fill them in.
- If you think you've said enough, don't feel like you have to pad it out. Short is good.

The problems vary a lot. Sometimes a single sentence will be an adequate answer. In other cases, a good presentation may take ten minutes or so. Hopefully everyone will get a chance to present a variety of different kinds of problems through the semester.

Collaboration

I encourage you to work together on exercises, but you are individually responsible for understanding, explaining, and defending anything you present in class. It's easy to trick yourself into thinking you understand something better than you do. This helps nobody. Here's a guideline if you work on a problem with others: set aside your notes from group sessions and write up your final solution on your own.

This does not apply to take-home exams. Do those by yourself.

Texts

Our main text will be notes that I am writing, and which will be posted online. This is a work in progress, and it will be evolving as the semester progresses. (So I don't recommend printing it all out!)

Here are some suggestions for some supplementary resources to look at, if you'd like another perspective. (If you come across other useful resources, please let me know!) On the course syllabus below I also list relevant parts of these textbooks for further reading.

- Sider, *Logic for Philosophy*
- Boolos, Burgess, and Jeffrey, *Computability and Logic*
- Enderton, *Mathematical Logic*

- The Open Logic Project: http://openlogicproject.org/download/

Tim Gowers has a series of blog posts on basic logical reasoning. These are oriented towards *using* logic in order to solve mathematical problems, and they may be helpful for getting the hang of solving the exercises.

https://gowers.wordpress.com/2011/10/09/basic-logic-summary/
https://gowers.wordpress.com/category/cambridge-teaching/basic-logic/

Outline of Topics

1. Sets and Functions
   - Every set has more subsets than it has elements. (Cantor's Theorem)

   See also Sider 1.8; Enderton 0.

2. The Infinite and the Uncountable
   - We can prove things about all numbers by taking for granted things about smaller numbers, and we can define functions on numbers using *recursion*.
   - There are just as many finite sequences as counting numbers.
   - There are more infinite sequences than counting numbers. (Cantor's Theorem Version 2.)
   - There are infinite sets with infinitely many different sizes.

   See also BBJ 1 & 2; Enderton 0.

3. Structures
   - We can represent operations in a structure using terms in a formal language. But no formal language with finitely many basic symbols can represent every operation in an infinite structure.

4. First-Order Logic
   - A review of the syntax and semantics of first-order quantifiers.

   See also Sider 4 & 5; BBJ 9 & 10; Enderton 2.0–2.2; B. Russell, "On Denoting."

5. The Inexpressible
   - No consistent theory can adequately describe itself. (Tarski's Theorem)
   - *See also* BBJ 17 & 15. Tarski, "The Semantic Conception of Truth and the Foundations of Semantics."

6. The Undecidable
   - There is no systematic method for finding out which methods will succeed. (Turing's Theorem)
   - There is no systematic method for finding out which sentences are consistent in first-order logic. (Church's Theorem)

   See also BBJ 6, 7, 4.1, 16; Enderton ch. 3.

7. The Unprovable
   - No theory is strong, simple, consistent, and complete. (Gödel's First Incompleteness Theorem)
   - No consistent theory proves its own consistency. (Gödel's Second Incompleteness Theorem)

See also BBJ 17, 18. Enderton ch. 3. Boolos, "Gödel's Second Incompleteness Theorem Explained in Words of One Syllable."

8. The Indescribable

   - Every argument in first-order logic has a proof or a counterexample, but not both. (Completeness)
   - No valid argument in first-order logic relies on infinitely many premises. (Compactness)
   - Any consistent theory in first-order logic is compatible with there only being countably many things. (The Löwenheim-Skolem Theorem)
   - "There are only finitely many F's" cannot be expressed in first-order logic.

   See also Sider 2.9; Enderton 2.5; BBJ ch. 13.

9. Second-Order Logic (if we have time)

   Second-order logic adds quantifiers for "properties" or "classes". Unlike first-order logic:

   - "There are only finitely many F's" can be expressed in second-order logic.
   - There are consistent second-order theories which imply that there are uncountably many things.
   - Second-order logic is incomplete: there is no system of formal proofs which can prove every valid argument of second-order logic without proving any invalid arguments.

Equality

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Students with Disabilities

Any student requesting academic accommodations based on a disability is required to register with Disability Services and Programs (DSP) each semester. A letter of verification for approved accommodations can be obtained from DSP. Please be sure the letter is delivered to me as early in the semester as possible.

Disability Services and Programs
Located in STU 301
Open 8:30am–5pm, Monday–Friday
213-740-0776 (Phone)
213-740-6948 (TTD)
ability@usc.edu
http://sait.usc.edu/academicsupport/centerprograms/dsp/home_index.html

Academic Integrity

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A: http://usc.edu/dept/publications/SCAMPUS/gov/ Students will be referred to the Office of Student Judicial Affairs and Community Standards for further review should there be any suspicion of academic dishonesty. The Review process can be found at: http://usc.edu/student-affaris/SJACS/ Information on intellectual property at USC is available at: http://usc.edu/academe/acsen/issues/ipr/index.html

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Changes

I may change anything at any time.